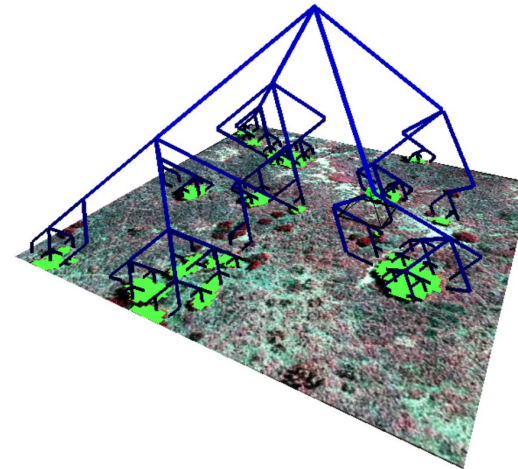
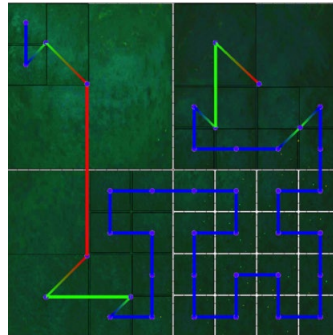
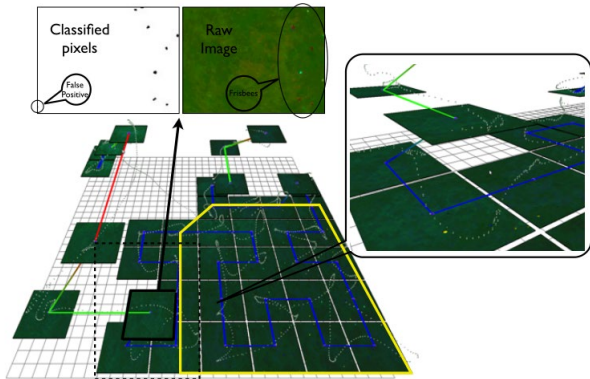




# Fractal Trajectories for Online Non-Uniform Aerial Coverage

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Autonomy Lab, Simon Fraser University  
{sas21, jwawerla, vaughan}@sfu.ca





Fractal Trajectories for Online Non-Uniform Aerial Coverage

在线非均匀空中覆盖的分形轨迹



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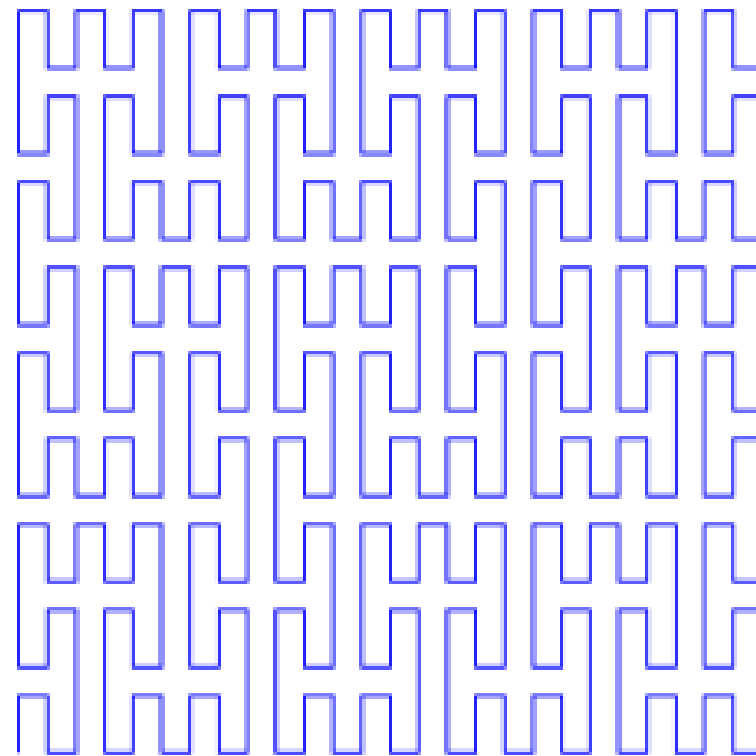
4、局限

## 背景知识介绍

**青年问禅师：“我的心被忧愁和烦恼塞满了怎么办？”**

禅师若有所思地说：“你随手画一条曲线。用放大镜放大了看。它的周围难道不是十分明朗开阔吗？”

那个青年画了一条皮亚诺曲线。



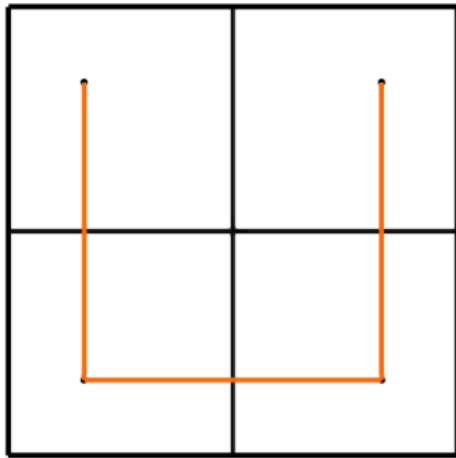


# 背景知识介绍

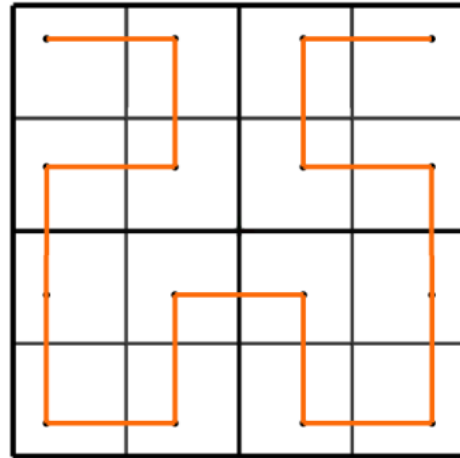
希尔伯特曲线:

The Hilbert curve (also known as the Hilbert space-filling curve) is a continuous fractal space-filling curve first described by the German mathematician David Hilbert in 1891, as a variant of the space-filling Peano curves discovered by Giuseppe Peano in 1890. ——Wikipedia

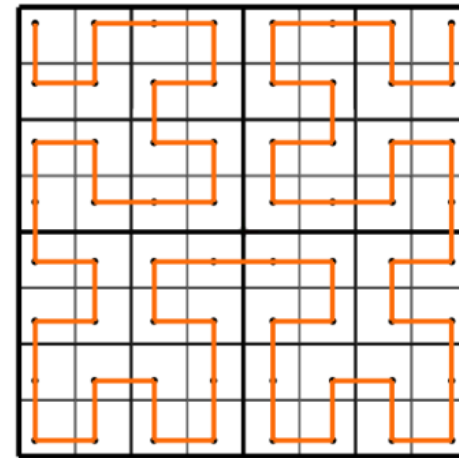
希尔伯特曲线（也称为希尔伯特空间填充曲线）是一种连续分形空间填充曲线，由德国数学家大卫·希尔伯特在1891年首次描述，是朱塞佩·皮亚诺在1890年发现的空间填充皮亚诺曲线的变体。——维基百科



1阶希尔伯特曲线



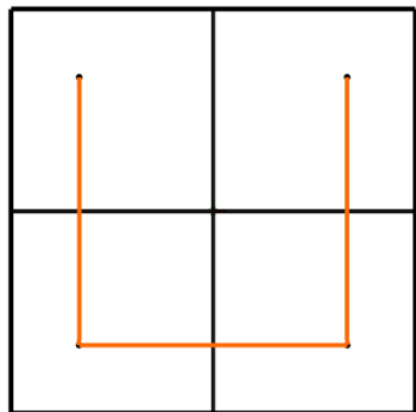
2阶希尔伯特曲线



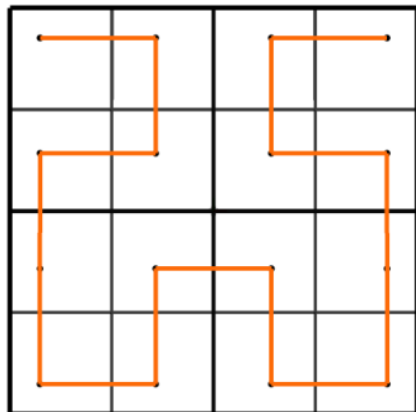
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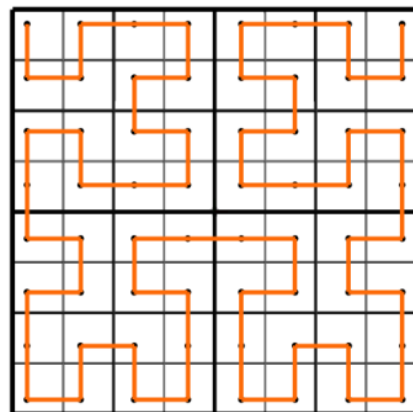
# 背景知识介绍



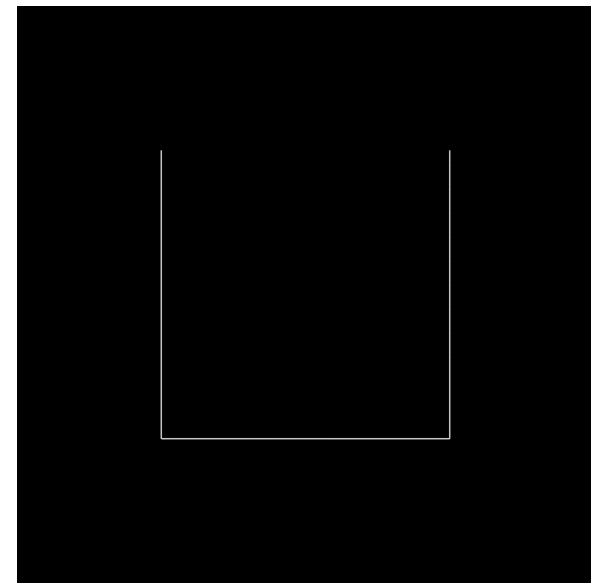
1阶希尔伯特曲线



2阶希尔伯特曲线



3阶希尔伯特曲线



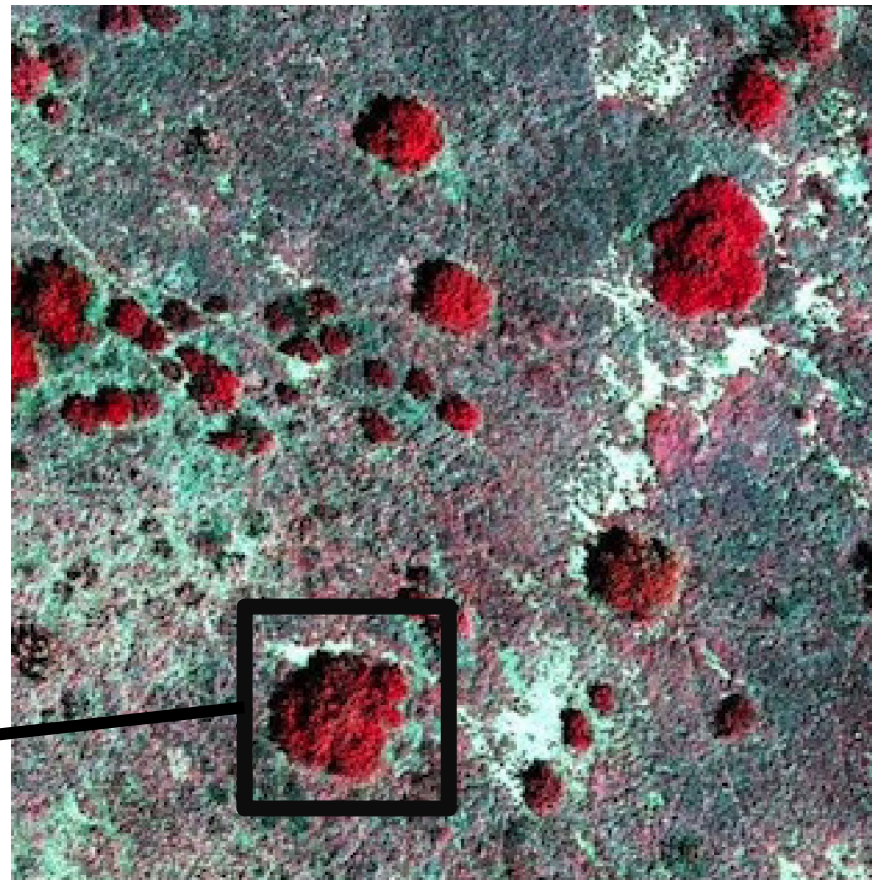




# 问题提出

- 在许多应用中，环境并不均匀，我们对目标区域的某一部分可能比其他部分更感兴趣。
- 无人机的所在的高度会影响所拍照片的分辨率。

某些我们更感兴趣的地方



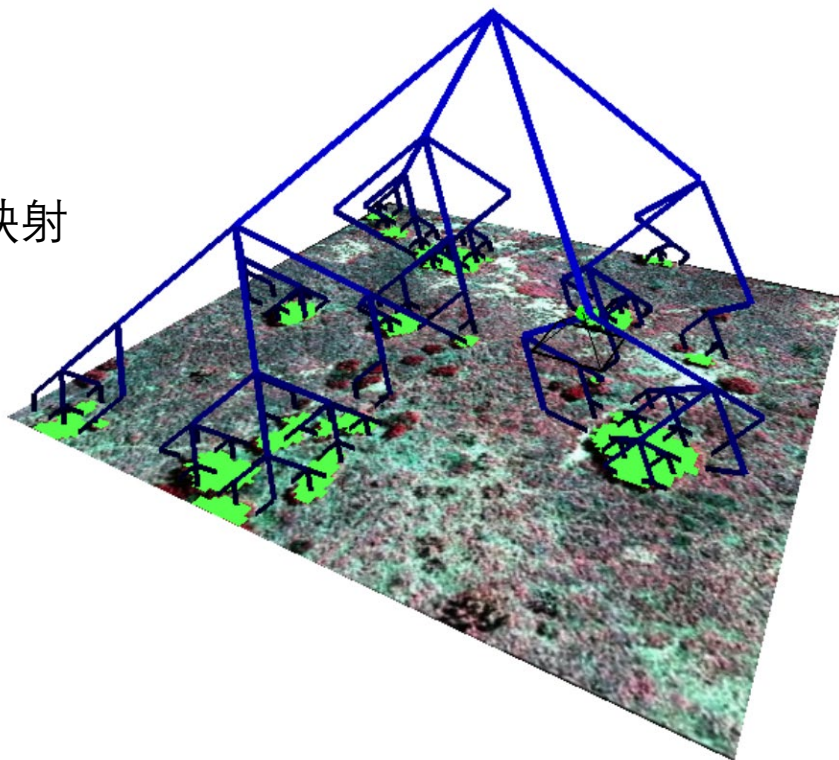
*Sadat, Seyed & Wawerla, Jens & Vaughan, Richard. (2014).  
Recursive non-uniform coverage of unknown terrains for UAVs.  
IEEE International Conference on Intelligent Robots and Systems.  
1742-1747. 10.1109/IROS.2014.6942790.*



# 算法思路

## 模型简化（覆盖树模型）：

- 我们需要覆盖的地方是一个正方形A(大小 $m \times m$ )
- 存在一些区域是我们更感兴趣的
- 函数 $l$ 定义为传感器所在高度到此传感器所覆盖区域的边长的映射
- 根R在A的正中间，高度 $h_r = l^{-1}(m)$
- $h_n$ 为节点 $n$ 的高度
- $A_n$ 为节点 $n$ 上传感器所能覆盖的面积
- 存在临界值 $h_t$ ，传感器高度不能低于这个值
- $A_n$ 被分成 $2 \times 2$ 的小格（cell）
- 每一个小格都可以有一个节点
- 访问父节点的时候，传感器总能知道那一部分是感兴趣的







# 算法思路

## 广度优先策略

- 遍历最高的节点
- 每到一个节点，就对其子节点加上标签（我们是否感兴趣）
- 结束上一层的遍历工作就进行下一层的遍历（不感兴趣的节点就不去了）

## 深度优先策略

- 每到一个节点，就把它底下所有感兴趣的节点走完，再去下一个节点

## 捷径启发策略 Shortcut Heuristic



# 算法思路

## 捷径启发策略

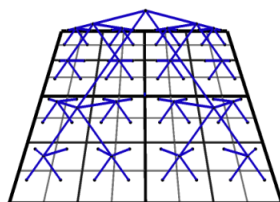
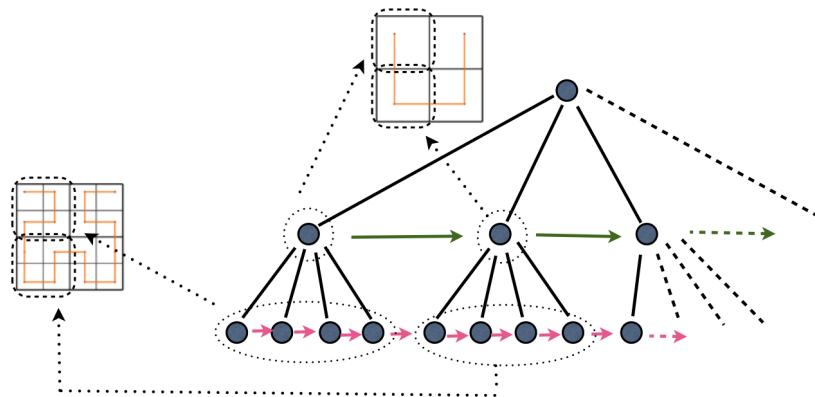
### Shortcut Heuristic

- 总体和深度优先一样
- 假设无人机从一个节点到下一个节点 ( $n \rightarrow n(\text{next})$ ) 且  $n(\text{next})$  的高度大于  $n$
- 无人机拜访  $n(\text{next})$  的子节点  $n(\text{nearest})$
- 如果  $n(\text{nearest})$  是有趣的, 拜访  $n(\text{next})$  的所有子节点
- 如果无趣, 无人机拜访  $n(\text{nearest})$

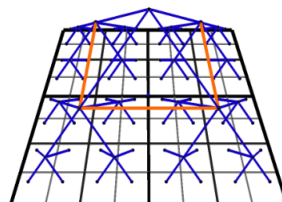


# 算法思路

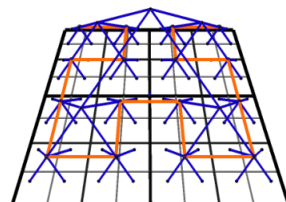
基于希尔伯特曲线的覆盖路径规划:



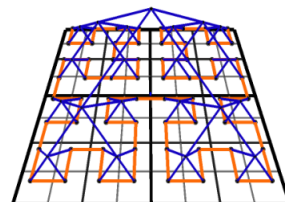
(a) Coverage tree



(b)  $H_1$



(c)  $H_2$



(d)  $H_3$

---

## Algorithm 1 Hilbert-based coverage path planning

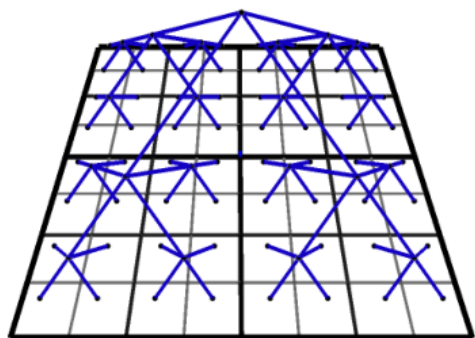
---

- 1: **Input:** Last visited node  $n$  (*null* if none), coverage tree  $T$  with the implicit order of the nodes.
  - 2: **Output:** Next node that should be visited or *null* if the traversal is finished.
  - 3: **if**  $n = \text{null}$  **then**  
    **return** the first leaf of  $T$
  - 4: **end if**
  - 5: **if**  $\text{Interesting}(n)$  AND  $\text{NeedVisit}(\text{Children}(n))$  **then**  
6:      $n \leftarrow$  first child of  $n$
  - 7: **else if**  $\text{!Interesting}(n)$  AND  $\text{Depth}(n) > 1$  **then**  
8:      $n \leftarrow \text{Parent}(n)$
  - 9: **end if**
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  - 11: **else if**  $n$  is the last child of  $\text{Parent}(n)$  **then**  
12:      $n \leftarrow \text{Parent}(n)$
  - 13:     Go to 5
  - 14: **else**  
15:     **if**  $\text{Next}(n) \neq \text{null}$  **then**  
16:          $n \leftarrow \text{Next}(n)$   
17:         Go to 10
  - 18:     **else**  
    **return** *null*
  - 19:     **end if**
  - 20: **end if**
-

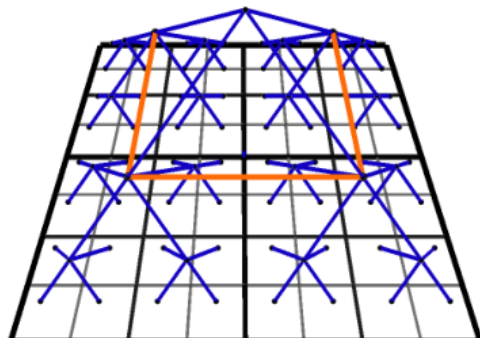


# 算法思路

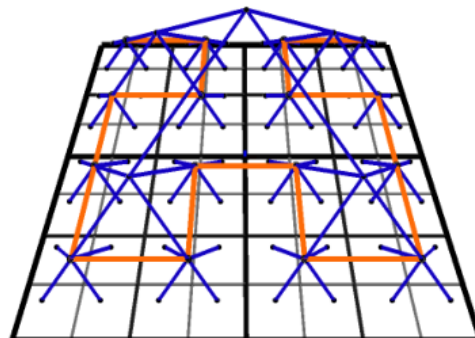
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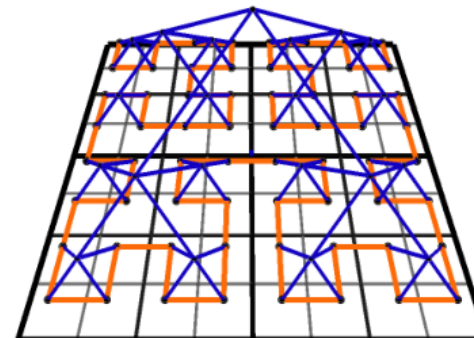
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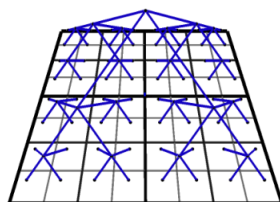
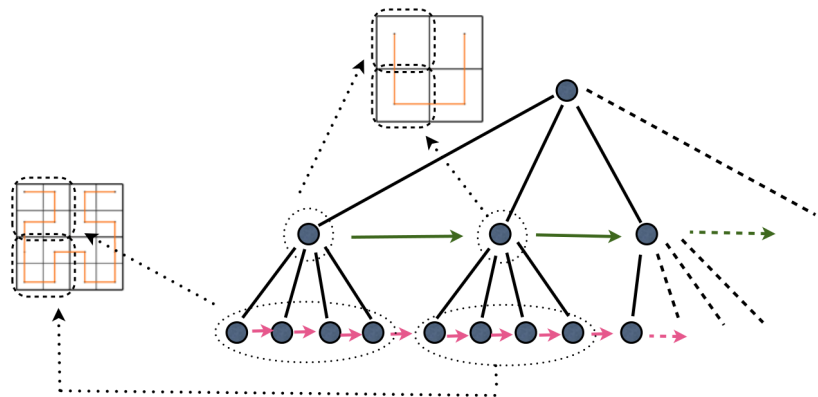


(d)  $H_3$

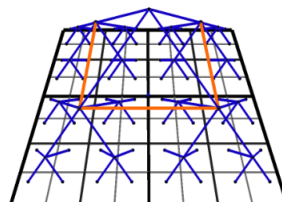


# 算法思路

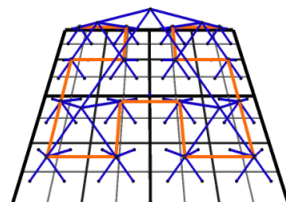
基于希尔伯特曲线的覆盖路径规划:



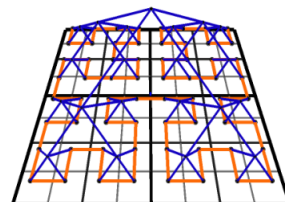
(a) Coverage tree



(b)  $H_1$



(c)  $H_2$



(d)  $H_3$

---

## Algorithm 1 Hilbert-based coverage path planning

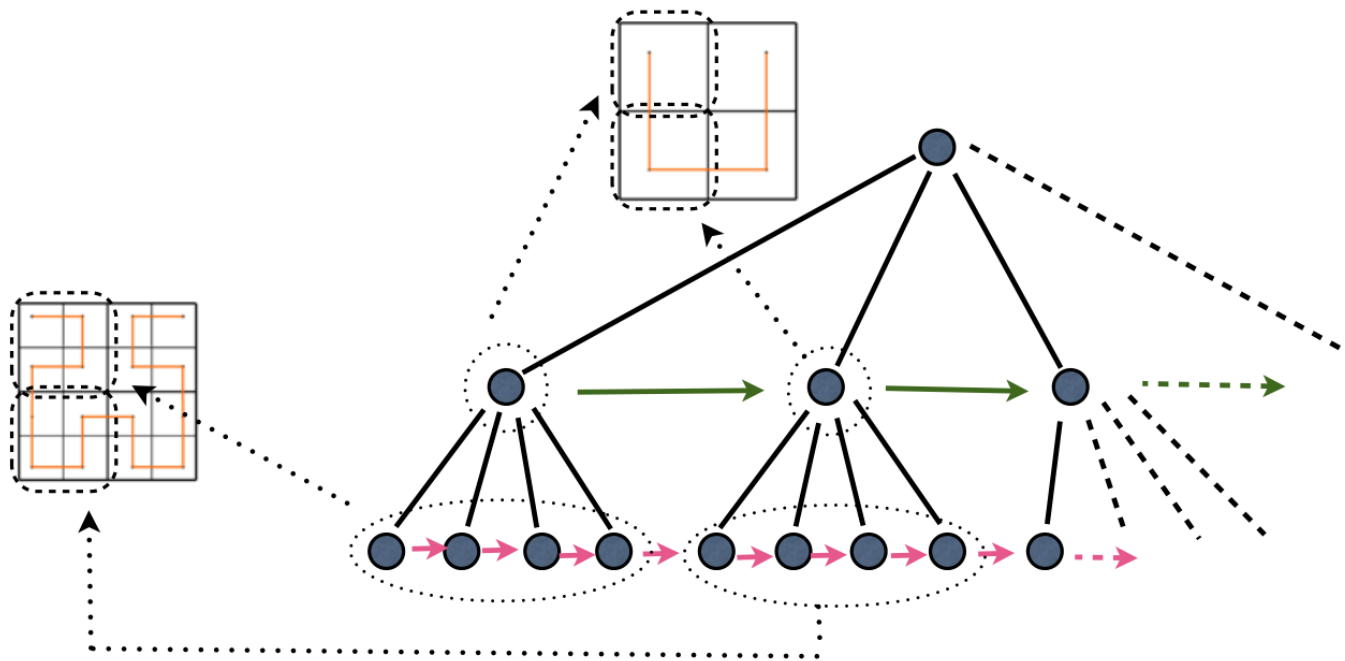
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  - 18:     **else**  
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# 算法思路

基于希尔伯特曲线的覆盖路径规划：

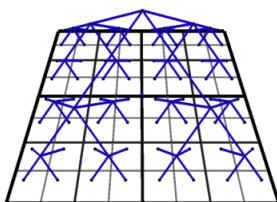
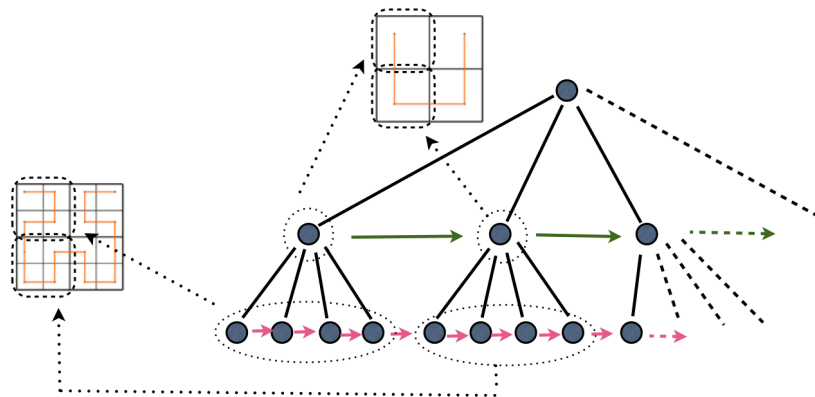




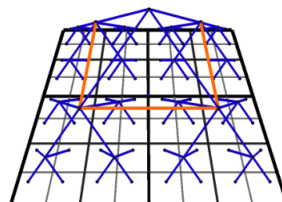


# 算法思路

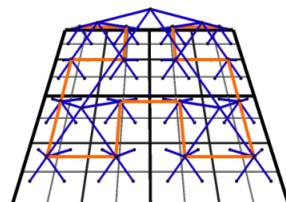
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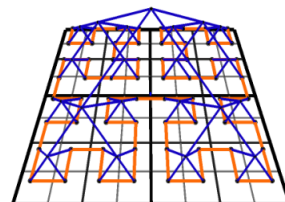
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---

**Algorithm 1** Hilbert-based coverage path planning

---

- 1: **Input:** Last visited node  $n$  (*null* if none), coverage tree  $T$  with the implicit order of the nodes.
  - 2: **Output:** Next node that should be visited or *null* if the traversal is finished.
  - 3: **if**  $n = \text{null}$  **then**  
    **return** the first leaf of  $T$
  - 4: **end if**
  - 5: **if**  $\text{Interesting}(n)$  AND  $\text{NeedVisit}(\text{Children}(n))$  **then**
  - 6:      $n \leftarrow$  first child of  $n$
  - 7: **else if**  $\text{!Interesting}(n)$  AND  $\text{Depth}(n) > 1$  **then**
  - 8:      $n \leftarrow \text{Parent}(n)$
  - 9: **end if**
  - 10: **if**  $\text{NeedVisit}(n)$  **then**  
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  - 12:      $n \leftarrow \text{Parent}(n)$
  - 13:     Go to 5
  - 14: **else**
  - 15:     **if**  $\text{Next}(n) \neq \text{null}$  **then**  
16:          $n \leftarrow \text{Next}(n)$   
17:         Go to 10
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# 算法思路

---

## Algorithm 1 Hilbert-based coverage path planning

---

- 1: **Input:** Last visited node  $n$  (*null* if none), coverage tree  $T$  with the implicit order of the nodes.
- 2: **Output:** Next node that should be visited or *null* if the traversal is finished.
- 3: **if**  $n = \text{null}$  **then**  
    **return** the first leaf of  $T$
- 4: **end if**



# 算法思路

```
5: if Interesting(n) AND NeedVisit(Children(n))  
   then  
6:   n ← first child of n  
7: else if !Interesting(n) AND Depth(n) > 1 then  
8:   n ← Parent(n)  
9: end if  
10: if NeedVisit(n) then  
    return n  
11: else if n is the last child of Parent(n) then  
12:   n ← Parent(n)  
13:   Go to 5  
14: else  
15:   if Next(n) <> null then  
16:     n ← Next(n)  
17:     Go to 10  
18:   else  
    return null  
19:   end if  
20: end if
```

---

在该算法中

*Children*(*n*)返回节点*n*的子节点

*Parent*(*n*)返回节点*n*的父节点

*Depth*(*n*)返回节点*n*所在树的深度

*NeedVisit*(*n*)返回true，如果访问*n*是必要的

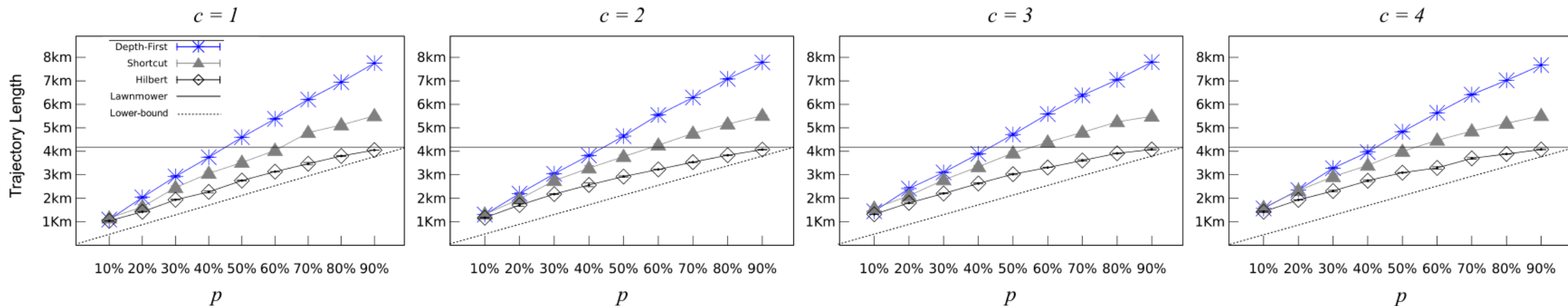


# 仿真结果

我们引入两个参数来表示有趣区域的分布:

P: 表示整个有趣区域的百分比

C: 表示有趣区域的数量



不同环境配置下的模拟实验结果

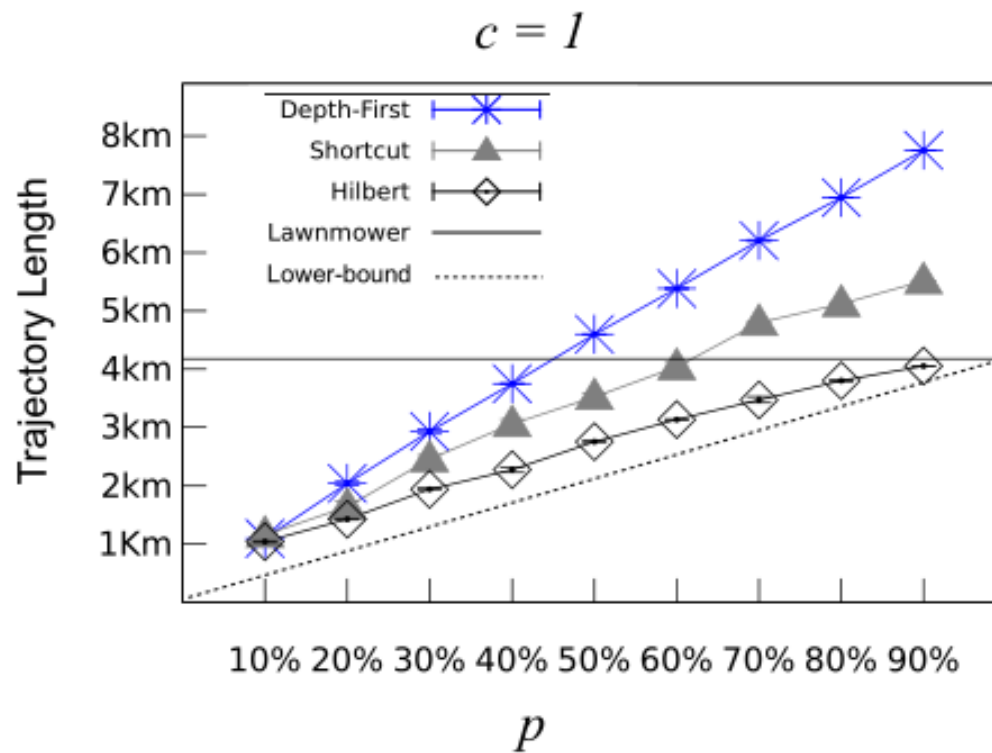
条件为:

总面积: 128m\*128m

传感器高度和覆盖边长的映射:  $l(h)=h$



# 仿真和实验结果





# 仿真和实验结果

为什么效果会更好呢？

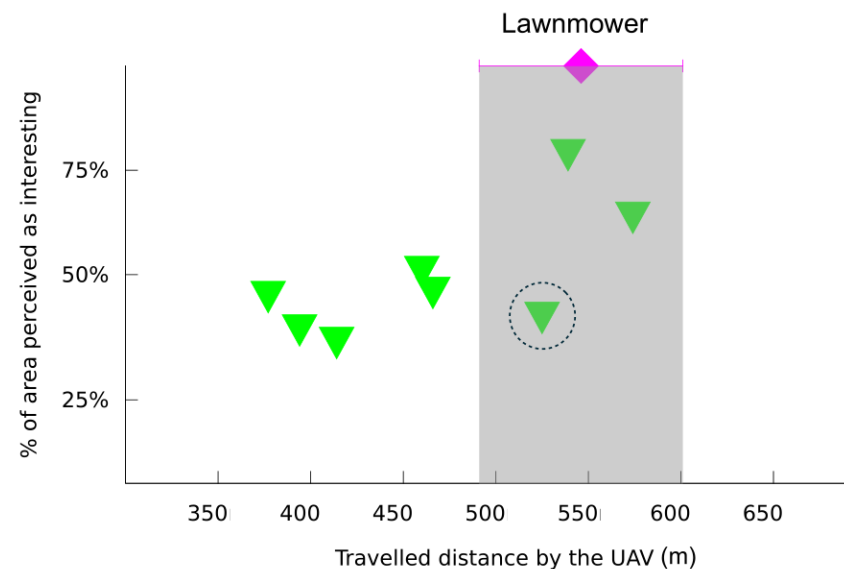




# 实验结果

作者使用升腾科技公司的鹈鹕四旋翼机在户外环境下进行了实验：

- 利用GPS估计机器人的位置，使用PID方法进行控制
- 任务是覆盖 $30 \times 30 \text{ m}^2$ 的区域
- 大量的飞盘分散在该区域，占目标环境的31%
- 彩色相机的 $I(h)=h$
- 查询区域与飞盘交集的图像被归类为有趣的
- 由于光线变化和其他噪声源，在飞盘检测中会出现误报
- 使用割草机的方法实验重复6次
- 使用作者提出的方法重复8次





# 局限

**和割草机策略相比：**

本文的方法需要更多地停顿，在速度上有劣势

关于z轴的移动成本，并不一定和平飞的成本相同